die Zeit ist tot
now, get to work

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ChaosBook.org/overheads/spatiotemporal
→ Chaotic field theory slides

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the goal

build
a chaotic field theory

from
the simplest chaotic blocks

using

- time invariance
- space invariance

of the defining partial differential equations

1. coin toss
2. kicked rotor
3. spatiotemporal cat
4. bye bye, dynamics

summary

spatiotemporal cat

insight 1 : how is turbulence described?

not by the evolution of an initial state

exponentially unstable system have finite (Lyapunov) time and space prediction horizons

but

by enumeration of admissible field configurations

and their natural weights

insight 2 : symbolic dynamics for turbulent flows

applies to all PDEs with *d* translational symmetries

a *d*-dimensional spatiotemporal field configuration

$$\{\phi_z\} = \{\phi_z, z \in \mathbb{Z}^d\}$$

is labelled by a *d*-dimensional spatiotemporal block of symbols

$$\{m_z\} = \{m_z, z \in \mathbb{Z}^d\},$$

rather than a single temporal symbol sequence

(as is done when describing a small coupled few-"body" system, or a small computational domain).

insight 3 : description of turbulence by invariant 2-tori

1 time, 0 space dimensions

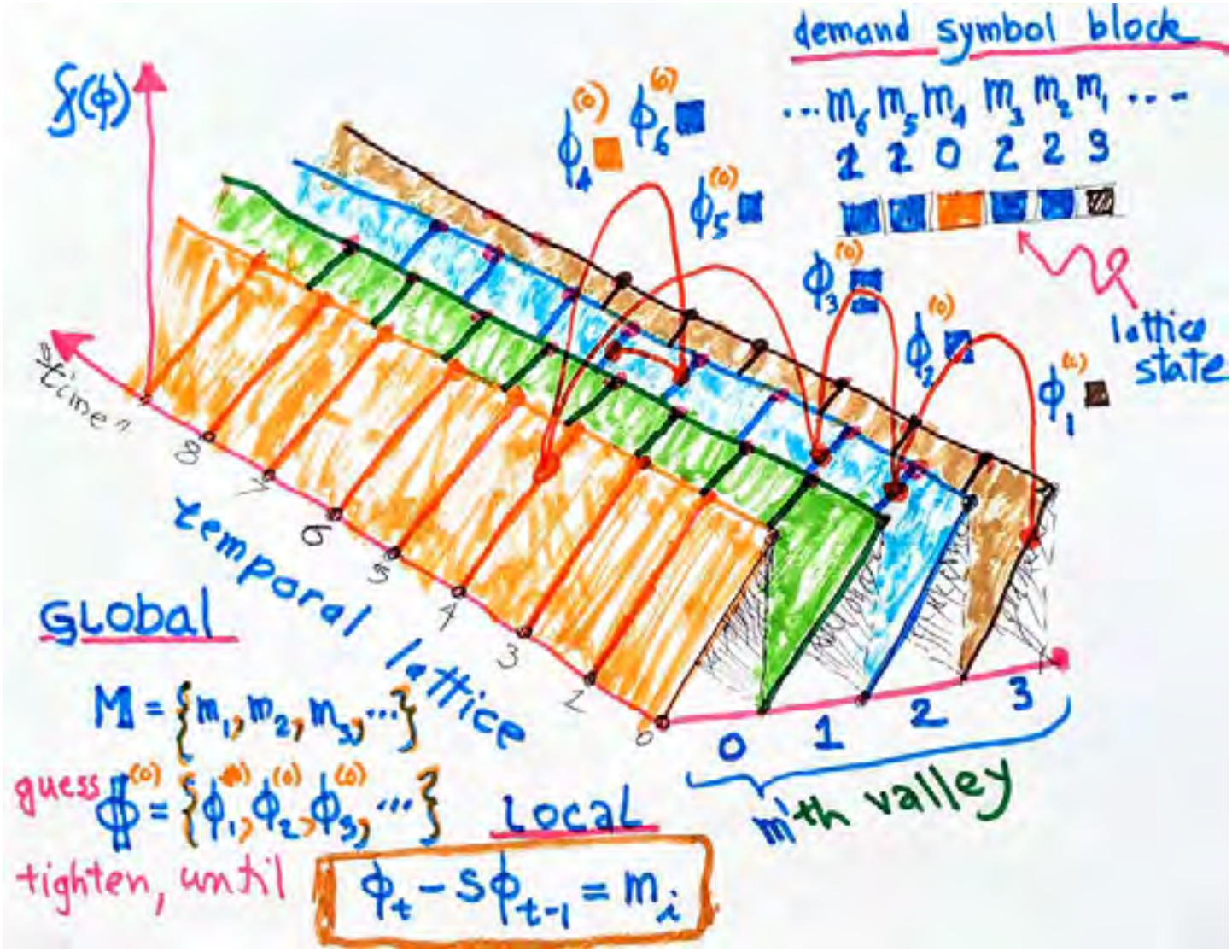
a phase space point is *periodic* if its orbit returns to itself after a finite time T; such orbit tiles the time axis by infinitely many repeats

1 time, d-1 space dimensions

a phase space point is *spatiotemporally periodic* if it belongs to an invariant d-torus $\mathcal{R}$,
i.e., a block $\mathrm{M}_{\mathcal{R}}$ that tiles the lattice state M,
with period ℓ_j in jth lattice direction

insight 4 : can compute ‘all’ solutions

Bernoulliland - rough initial guesses converge



no exponential instabilities

what we still do not understand today

1. solved so far only 1-dimensional spatiotemporal lattice, point group D_1
2. should all time-reversal symmetric systems be analyzed this way ?
3. should all dynamical systems should be solved on reciprocal lattice ?
4. for 2-dimensional spatiotemporal chaotic field theory, still have to do this for square lattice point group D_4
5. then, solve the problem of turbulence (Navier-Stokes, Yang-Mills, general relativity)

die Zeit ist tot

also, an die Arbeit!

bye bye, dynamics

1. goal : describe states of turbulence in infinite spatiatemporal domains
2. theory : classify, enuremate all spatiotemporal tilings
3. example : spatiotemporal cat, the simplest model of “turbulence”

there is no more time

there is only enumeration of admissible spacetime field configurations

time is dead

now, get to work

goodbye

to long time and/or space integrators

they never worked and could never work

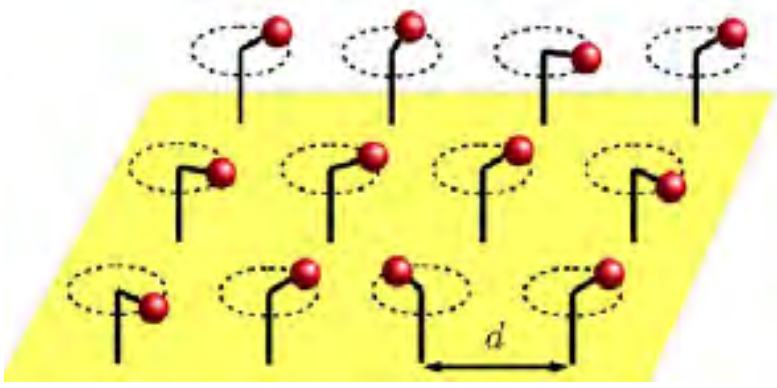
the stage is set for the quantum field theory of spatiotemporal cat, the details of which we leave to our always trustworthy friends Jon Keating and Marcos Saraceno

take-home :

traditional field theory

Helmholtz

chaotic field theory



damped Poisson, Yukawa